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# Principal filters of some ordered Γ-semigroups

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**Abstract.** For an intra-regular or a left regular and left duo ordered Γ-semigroup $M$, we describe the principal filter of $M$ which plays an essential role in the structure of this type of *po*-Γ-semigroups. We also prove that an ordered Γ-semigroup $M$ is intra-regular if and only if the ideals of $M$ are semiprime and it is left (right) regular and left (right) duo if and only if the left (right) ideals of $M$ are semiprime.

*Key Words:* ordered Γ-semigroup, filter, intra-regular, left regular
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## 1 Introduction and prerequisites

Our aim is to describe the principal filters of intra-regular ordered Γ-semigroups and the principal filters of ordered Γ-semigroups which are both left regular and left duo. Croisot, who used the term "inversive" instead of "regular", connects the matter of decomposition of a semigroup with the regularity and semiprime conditions [2]. A semigroup $S$ is said to be left (resp. right) regular if for every $a \in S$ there exists $x \in S$ such that $a = xa^2$ (resp. $a = a^2x$). That is, if $a \in Sa^2$ (resp. $a \in a^2S$) for every $a \in S$ which is equivalent to saying that $A \subseteq A^2S$ (resp. $A \subseteq SA^2$) for every $A \subseteq S$. A semigroup $S$ is said to be intra-regular if for every $a \in S$ there exist $x, y \in S$ such that $a = xa^2y$. In other words, if $a \in Sa^2S$ for every $a \in S$ or $A \subseteq SA^2S$ for every $A \subseteq S$. For decompositions of an intra-regular, of a left regular or both left and right regular semigroup we refer to [1, 7]. The concepts of intra-regular ordered semigroup and of right regular ordered semigroup have been introduced in [3, 4] in which the decomposition of an intra-regular ordered semigroup into simple components and the decomposition of a right regular and right duo ordered semigroup into right simple components have been studied. The principal filter of $S$ has a very simple form for both ordered and unordered case of Γ-semigroups, and it plays an essential role in their decomposition.

For the sake of completeness, let us first give the definition of a Γ-semigroup. In this paper we use the definition of Γ-semigroup introduced by Saha in [8]: Given two nonempty sets $M$ and $\Gamma$, $M$ is called a Γ-semigroup if there exists a mapping $M \times \Gamma \times M \to M \mid (a,\gamma,b) \to a\gamma b$ such that $(a\gamma b)\mu c = a\gamma(b\mu c)$ for every $a,b,c \in M$ and every $\gamma,\mu \in \Gamma$. An *ordered* Γ-*semigroup* (shortly, *po*-Γ-*semigroup*) is clearly a Γ-semigroup $M$ with an order relation "$\leq$" on $M$ such that $a \leq b$ implies $a\gamma c \leq b\gamma c$ and $c\gamma a \leq c\gamma b$ for every $c \in M$ and every $\gamma \in \Gamma$. For a subset $H$ of $M$ we denote by $(H]$ the subset of $M$ defined by

$$(H] = \{t \in M \mid t \leq a \text{ for some } t \in H\}.$$

We mention the properties we use in the paper: Clearly $M = (M]$, and for any subsets $A,B,C$ of $M$, we have the following: $A \subseteq (A]$; if $A \subseteq B$, then $A\Gamma C \subseteq B\Gamma C$ and $C\Gamma A \subseteq C\Gamma B$; if $A \subseteq B$, then $(A] \subseteq (B]$; $(A]\Gamma(B] \subseteq (A\Gamma B]$; $\Big((A]\Gamma(B]\Big] = \Big((A]\Gamma B\Big] = \Big(A\Gamma(B]\Big] = (A\Gamma B]$; if $a \leq b$, then $A\Gamma a \subseteq (A\Gamma b]$ and $a\Gamma A \subseteq (b\Gamma A]$; $\Big((A]\Big] = (A]$. Let us prove the last one: Since $A \subseteq (A]$, we have $(A] \subseteq \Big((A]\Big]$. Let now $t \in \Big((A]\Big]$. Then $t \leq x$ for some $x \in (A]$ and $x \leq a$ for some $a \in A$. Since $t \in S$ and $t \leq a$, where $a \in A$, we have $t \in (A]$. As one can easily see, the following are equivalent: (1) $a \in A$ and $S \ni b \leq a$, then $b \in A$. (2) $(A] \subseteq A$. (3) $(A] = A$. A nonempty subset $A$ of $M$ is called a *subsemigroup* of $M$ if, for every $a,b \in A$ and every $\gamma \in \Gamma$, we have $a\gamma b \in A$, that is if $A\Gamma A \subseteq A$. A nonempty subset $A$ of $M$ is called a *left* (resp. *right*) *ideal* of $M$ if (1) $M\Gamma A \subseteq A$ (resp. $A\Gamma M \subseteq A$) and (2) if $a \in A$ and $M \ni b \leq a$, then $b \in A$ (equivalently $(A] = A$, which in turn is equivalent to $(A] = A$). It is called an *ideal* (or *two-sided ideal*) of $M$ if it is both a left and right ideal of $M$. Clearly every left (resp. right) ideal of $M$ is a subsemigroup of $M$. A *po*-Γ-semigroup $M$ is called *left* (resp. *right*) *duo* if the left (resp. right) ideals of $M$ are two-sided. A subsemigroup $F$ of $M$ is called a *filter* of $M$ if (1) for every $a,b \in M$ and every $\gamma \in \Gamma$ such that $a\gamma b \in F$, we have $a \in F$ and $b \in F$ and (2) if $a \in F$ and $M \ni b \geq a$, then $b \in F$. For an element $x$ of $M$, we denote by $N(x)$ *the filter of* $M$ *generated by* $x$ (that is, the least with respect to the inclusion relation filter of $M$ containing $x$). A subset $T$ of $M$ is called *semiprime* if $x \in M$ and $\gamma \in \Gamma$ such that $x\gamma x \in T$ implies $x \in T$.

As we know, many results on semigroups (ordered semigroups) can be transferred into Γ-semigroups (*po*-Γ-semigroups) just putting a Gamma in the appropriate place, while for some other results the transfer needs subsequent technical changes. A Γ-semigroup $M$ is called *intra-regular* if $a \in M\Gamma a\Gamma a\Gamma M$ for every $a \in M$, equivalently if $A \subseteq M\Gamma A\Gamma A\Gamma M$ for every $A \subseteq M$. It is called *left* (resp. *right*) *regular* if $a \in M\Gamma a\Gamma a$ (resp. $a \in a\Gamma a\Gamma M$) for every $a \in M$, equivalently if $A \subseteq M\Gamma A\Gamma A$ (resp. $A \subseteq A\Gamma A\Gamma M$) for every $A \subseteq M$. An ordered $\Gamma$-semigroup $M$ is called *intra-regular* if for every $a \in M$ we have $a \in (M\Gamma a\Gamma a\Gamma M]$, equivalently if for every $A \subseteq M$ we have $A \subseteq (M\Gamma A\Gamma A\Gamma M]$. It is called *left* (resp. *right*) *regular* if $a \in (M\Gamma a\Gamma a]$ (resp. $(a \in (a\Gamma a\Gamma M])$ for every $a \in M$, equivalently if $A \subseteq (M\Gamma A\Gamma A]$ (resp. $A \subseteq (A\Gamma A\Gamma M])$ for every $A \subseteq M$. Although some interesting results on $\Gamma$-semigroups are obtained using the definition of left (resp. right) regular or the definition of intra-regular ordered $\Gamma$-semigroup mentioned above, with these definitions one fails to prove basic results of $\Gamma$-semigroups, such as to describe the filter of $M$ generated by an element $a$ of $M$, for example, which plays an essential role in the investigation. To overcome this difficulty, a new definition of intra-regular and a new definition of left regular $\Gamma$-semigroups has been introduced in [5]. The intra-regular $\Gamma$-semigroup has been defined as a $\Gamma$-semigroup $M$ such that $a \in M\Gamma a\gamma a\Gamma M$ for each $a \in M$ and each $\gamma \in \Gamma$ and the left (resp. right) regular $\Gamma$-semigroup as a $\Gamma$-semigroup in which $a \in M\Gamma a\gamma a$ (resp. $a \in a\gamma a\Gamma M$) for each $a \in M$ and each $\gamma \in \Gamma$ and it is proved that a $\Gamma$-semigroup $M$ is left regular (in that new sense) if and only if it is a union of a family of left simple subsemigroups on $M$. And in [6] we gave some further structure theorems of this type of $\Gamma$-semigroups using that new definition and the form of their principal filters. But what happens in case of intra-regular or in case of left regular or for right regular *po*-$\Gamma$-semigroups? Can we describe the form of their principal filters using some new definitions similar to the unordered case? The present paper gives the related answer.

## 2 On intra-regular ordered *po*-$\Gamma$-semigroups

We characterize here the intra-regular *po*-$\Gamma$-semigroups in terms of filters, and we prove that a *po*-$\Gamma$-semigroup $M$ is intra-regular if and only if the ideals of $M$ are semiprime.

**Definition 1.** An ordered $\Gamma$-semigroup $M$ is called *intra-regular* if

$$x \in (M\Gamma x\gamma x\Gamma M]$$

for every $x \in M$ and every $\gamma \in \Gamma$.

**Definition 2.** (cf. also [5]) If $M$ is an ordered $\Gamma$-semigroup, a subset $A$ of $M$ is called *semiprime* if

$$a \in M \text{ and } \gamma \in \Gamma \text{ such that } a\gamma a \in A \text{ implies } a \in A.$$

**Theorem 3.** *An ordered $\Gamma$-semigroup $M$ is intra-regular if and only if, for every $x \in M$, we have*

$$N(x) = \{y \in M \mid x \in (M\Gamma y\Gamma M]\}.$$

**Proof.** $\Longrightarrow$. Let $x \in M$ and $T := \{y \in M \mid x \in (M\Gamma y\Gamma M]\}$. Then we have the following:

(1) $T$ is a nonempty subset of $M$. Indeed: Take an element $\gamma \in \Gamma$ $(\Gamma \neq \emptyset)$. Since $M$ is intra-regular, we have

$$x \in (M\Gamma x\gamma x\Gamma M] = \Big((M\Gamma x)\gamma x\Gamma M\Big] \subseteq \Big((M\Gamma M)\Gamma x\Gamma M\Big] \subseteq (M\Gamma x\Gamma M],$$

so $x \in T$.

(2) Let $a, b \in T$ and $\gamma \in \Gamma$. Then $a\gamma b \in T$. Indeed: Since $a \in T$, we have $x \in (M\Gamma a\Gamma M]$. Since $b \in T$, we have $x \in (M\Gamma b\Gamma M]$. Since $M$ is intra-regular, $x \in M$ and $\gamma \in \Gamma$, we have $x \in (M\Gamma x\gamma x\Gamma M]$. Then we have

$$\begin{aligned} x \in (M\Gamma x\gamma x\Gamma M] &\subseteq \Big(M\Gamma(M\Gamma b\Gamma M]\gamma(M\Gamma a\Gamma M]\Gamma M\Big] \\ &= \Big(M\Gamma(M\Gamma b\Gamma M)\gamma(M\Gamma a\Gamma M)\Gamma M\Big] \\ &= \Big((M\Gamma M)\Gamma(b\Gamma M\gamma M\Gamma a)\Gamma(M\Gamma M)\Big] \\ &\subseteq \Big(M\Gamma(b\Gamma M\gamma M\Gamma a)\Gamma M\Big]. \end{aligned}$$

We prove that $b\Gamma M\gamma M\Gamma a \subseteq \Big(M\Gamma(a\gamma b)\Gamma M\Big]$. Then we have

$$\begin{aligned} x &\in \Big(M\Gamma\Big(M\Gamma(a\gamma b)\Gamma M\Big]\Gamma M\Big] = \Big(M\Gamma\Big(M\Gamma(a\gamma b)\Gamma M\Big)\Gamma M\Big] \\ &= \Big((M\Gamma M)\Gamma(a\gamma b)\Gamma(M\Gamma M)\Big] \subseteq \Big(M\Gamma(a\gamma b)\Gamma M\Big], \end{aligned}$$

so $a\gamma b \in T$. Let now $b\lambda u\gamma v\delta a \in b\Gamma M\gamma M\Gamma a$ for some $u, v \in M$, $\lambda, \delta \in \Gamma$. Since $M$ is intra-regular, for the elements $b\lambda u\gamma v\delta a \in M$ and $\gamma \in \Gamma$, we have

$$\begin{aligned} b\lambda u\gamma v\delta a &\in \Big(M\Gamma(b\lambda u\gamma v\delta a)\gamma(b\lambda u\gamma v\delta a)\Gamma M\Big] \\ &= \Big((M\Gamma b\lambda u\gamma v)\delta(a\gamma b)\lambda(u\gamma v\delta a\Gamma M)\Big] \\ &\subseteq \Big(M\Gamma(a\gamma b)\Gamma M\Big]. \end{aligned}$$

(3) Let $a, b \in M$ and $\gamma \in \Gamma$ such that $a\gamma b \in T$. Then $a, b \in T$. Indeed: Since $a\gamma b \in T$, we have $x \in \Big(M\Gamma(a\gamma b)\Gamma M\Big] \subseteq \Big(M\Gamma a\gamma(M\Gamma M)\Big] \subseteq (M\Gamma a\Gamma M]$, so $a \in T$. Since $x \in \Big(M\Gamma(a\gamma b)\Gamma M\Big] \subseteq \Big((M\Gamma M)\gamma b\Gamma M\Big] \subseteq (M\Gamma b\Gamma M]$, we have $b \in T$.

(4) Let $a \in T$ and $M \ni b \geq a$. Then $b \in T$. Indeed: Since $a \in T$, we have $x \in (M\Gamma a\Gamma M]$. Since $a \leq b$, we have $M\Gamma a\Gamma M \subseteq (M\Gamma b\Gamma M]$, then $(M\Gamma a\Gamma M] \subseteq \Big((M\Gamma b\Gamma M]\Big] = (M\Gamma b\Gamma M]$. Then we have $x \in (M\Gamma b\Gamma M]$, and $b \in T$.

(5) Let $F$ be a filter of $M$ such that $x \in F$. Then $T \subseteq F$. Indeed: Let $a \in T$. Then $x \in (M\Gamma a\Gamma M]$, so $F \ni x \le u\lambda(a\mu v)$ for some $u, v \in M$, $\lambda, \mu \in \Gamma$. Since $F$ is a filter of $M$, $x \in F$ and $M \ni u\lambda(a\mu v) \ge x$, we have $u\lambda(a\mu v) \in F$. Since $F$ is a filter of $M$, $u, a\mu v \in M$, $\lambda \in \Gamma$ and $u\lambda(a\mu v) \in F$, we have $a\mu v \in F$, again since $F$ is a filter of $M$, $a, v \in M$ and $\mu \in \Gamma$, we have $a \in F$.

$\Longleftarrow$. Let $x \in M$ and $\gamma \in \Gamma$. Then $x \in (M\Gamma x\gamma x\Gamma M]$. Indeed: Since $N(x)$ is a subsemigroup of $M$, $x \in N(x)$ and $\gamma \in \Gamma$, we have $x\gamma x \in N(x)$. Then, by hypothesis, we get $x \in \Big(M\Gamma(x\gamma x)\Gamma M\Big] = (M\Gamma x\gamma x\Gamma M]$, thus $M$ is intra-regular. □

**Theorem 4.** *An ordered* $\Gamma$*-semigroup* $M$ *is intra-regular if and only if the ideals of* $M$ *are semiprime.*

**Proof.** $\Longrightarrow$. Let $A$ be an ideal of $M$, $x \in M$ and $\gamma \in \Gamma$ such that $x\gamma x \in A$. Since $M$ is intra-regular, we have

$$x \in \Big(M\Gamma(x\gamma x)\Gamma M\Big] \subseteq \Big((M\Gamma A)\Gamma M\Big] \subseteq (A\Gamma M] \subseteq (A] = A,$$

then $x \in A$, and $A$ is semiprime.

$\Longleftarrow$. Let $x \in M$ and $\gamma \in \Gamma$. Then $x \in (M\Gamma x\gamma x\Gamma M]$. In fact: The set $(M\Gamma x\gamma x\Gamma M]$ is an ideal of $M$. This is because it is a nonempty subset of $M$, $M\Gamma(M\Gamma x\gamma x\Gamma M] \subseteq \Big(M\Gamma(M\Gamma x\gamma x\Gamma M]\Big] = \Big(M\Gamma(M\Gamma x\gamma x\Gamma M)\Big] \subseteq (M\Gamma x\gamma x\Gamma M]$, $(M\Gamma x\gamma x\Gamma M]\Gamma M \subseteq (M\Gamma x\gamma x\Gamma M]$, and $\Big((M\Gamma x\gamma x\Gamma M]\Big] = (M\Gamma x\gamma x\Gamma M]$ (since this holds for any subset $A$ of $M$). Since $(M\Gamma x\gamma x\Gamma M]$ is semiprime, $x\gamma x \in M\Gamma M \subseteq M$, $\gamma \in \Gamma$ and

$$(x\gamma x)\gamma(x\gamma x) = x\gamma(x\gamma x)\gamma x \in M\Gamma x\gamma x\Gamma M \subseteq (M\Gamma x\gamma x\Gamma M],$$

we have $x\gamma x \in (M\Gamma x\gamma x\Gamma M]$. Then, since $x \in M$, $\gamma \in \Gamma$ and $(M\Gamma x\gamma x\Gamma M]$ is semiprime, we have $x \in (M\Gamma x\gamma x\Gamma M]$, so $M$ is intra-regular. □

# 3 On left regular and left duo *po*-$\Gamma$-semigroups

First we notice that the left (and the right) regular *po*-$\Gamma$-semigroups are intra-regular. Then we characterize the *po*-$\Gamma$-semigroups which are both left regular and left duo in terms of filters and we prove that a *po*-$\Gamma$-semigroup $M$ is left (resp. right) regular if and only if the left (resp. right) ideals of $M$ are semiprime.

**Definition 5.** An ordered $\Gamma$-semigroup $M$ is called *left regular* (resp. *right regular*) if

$$x \in (M\Gamma x\gamma x] \text{ (resp. } x \in (x\gamma x\Gamma M])$$

for every $x \in M$ and every $\gamma \in \Gamma$.

**Proposition 6.** *Let $M$ be an ordered $\Gamma$-semigroup. If $M$ is left (resp. right) regular, then $M$ is intra-regular.*

**Proof.** Let $M$ be left regular, $x \in M$ and $\gamma \in \Gamma$. Then we have

$$\begin{aligned} x \in (M\Gamma x\gamma x] &\subseteq \Big(M\Gamma(M\Gamma x\gamma x]\gamma x\Big] = \Big(M\Gamma(M\Gamma x\gamma x)\gamma x\Big] \\ &\subseteq \Big((M\Gamma M)\Gamma(x\gamma x)\Gamma M\Big] \subseteq \Big(M\Gamma x\gamma x\Gamma M\Big], \end{aligned}$$

thus $M$ is intra-regular. Similarly, the right regular *po*-$\Gamma$-semigroups are intra-regular. □

**Theorem 7.** *An ordered $\Gamma$-semigroup $M$ is left regular and left duo if and only if, for every $x \in M$, we have*

$$N(x) = \{y \in M \mid x \in (M\Gamma y]\}.$$

**Proof.** $\Longrightarrow$. Let $x \in M$ and $T := \{y \in M \mid x \in (M\Gamma y]\}$. Since $M$ is left regular, we have $x \in (M\Gamma x\gamma x] \subseteq \Big((M\Gamma M)\Gamma x\Big] \subseteq (M\Gamma x]$, so $x \in T$, and $T$ is a nonempty subset of $M$.

Let $a, b \in T$ and $\gamma \in \Gamma$. Since $x \in (M\Gamma a]$, $x \in (M\Gamma b]$ and $M$ is left regular, we have

$$\begin{aligned} x \in (M\Gamma x\gamma x] &\subseteq \Big(M\Gamma(M\Gamma b]\gamma(M\Gamma a]\Big] = \Big(M\Gamma(M\Gamma b)\gamma(M\Gamma a)\Big] \\ &\subseteq \Big(M\Gamma(b\gamma M\Gamma a)\Big]. \end{aligned}$$

In addition, $b\gamma M\Gamma a \subseteq (M\Gamma a\gamma b]$. Indeed: Let $b\gamma u\mu a \in b\gamma M\Gamma a$, where $u \in M$ and $\mu \in \Gamma$. Since $M$ is left regular, we have

$$b\gamma u\mu a \in \Big(M\Gamma(b\gamma u\mu a)\gamma(b\gamma u\mu a)\Big] \subseteq \Big(M\Gamma(a\gamma b)\Gamma M\Big] = \Big((M\Gamma a\gamma b]\Gamma M\Big].$$

Since $(M\Gamma a\gamma b]$ is a left ideal of $M$, it is a right ideal of $M$ as well, so $(M\Gamma a\gamma b]\Gamma M \subseteq (M\Gamma a\gamma b]$, then $b\gamma u\mu a \in \Big((M\Gamma a\gamma b]\Big] = (M\Gamma a\gamma b]$. Hence we obtain

$$x \in \Big(M\Gamma(M\Gamma a\gamma b]\Big] = \Big(M\Gamma(M\Gamma a\gamma b)\Big] \subseteq \Big(M\Gamma(a\gamma b)\Big],$$

from which $a\gamma b \in T$.

Let $a, b \in M$ and $\gamma \in \Gamma$ such that $a\gamma b \in T$. Since $x \in (M\Gamma a\gamma b] \subseteq (M\Gamma b]$, we have $b \in T$. Besides, $x \in (M\Gamma a\gamma b] \subseteq \Big((M\Gamma a]\Gamma M\Big]$. The set $(M\Gamma a]$ as a left ideal of $M$, it is a right ideal of $M$ as well, so $(M\Gamma a]\Gamma M \subseteq (M\Gamma a]$. Thus we have $x \in \Big((M\Gamma a]\Big] = (M\Gamma a]$, and $a \in T$.

Let $a \in T$ and $M \ni b \geq a$. Then we have $x \in (M\Gamma a] \subseteq (M\Gamma b]$, so $b \in T$.

Let $F$ be a filter of $M$ such that $x \in F$ and let $a \in T$. Since $x \in (M\Gamma a]$, we have $F \ni x \leq u\mu a$ for some $u \in M$, $\mu \in \Gamma$. Since $F$ is a filter of $M$, we have $u\mu a \in F$, and $a \in F$.

$\Longleftarrow$. Let $x \in M$ and $\gamma \in \Gamma$. Since $x \in N(x)$ and $N(x)$ is a subsemigroup of $M$, we have $x\gamma x \in N(x)$. By hypothesis, we get $x \in (M\Gamma x\gamma x]$, so $M$ is left regular. Let now $A$ be a left ideal of $M$, $a \in A$, $\gamma \in \Gamma$ and $u \in M$. Since $a\gamma u \in N(a\gamma u)$ and $N(a\gamma u)$ is a filter of $M$, we have $a \in N(a\gamma u)$. By hypothesis, we have $a\gamma u \in (M\Gamma a] \subseteq (M\Gamma A] \subseteq (A] = A$. Thus $A$ is right ideal of $M$, and $M$ is left duo. □

The right analogue of Theorem 7 also holds, and we have

**Theorem 8.** *An ordered $\Gamma$-semigroup $M$ is right regular and right duo if and only if, for every $x \in M$, we have*

$$N(x) = \{y \in M \mid x \in (y\Gamma M]\}.$$

**Theorem 9.** *An ordered $\Gamma$-semigroup $M$ is left (resp. right) regular if and only if the left (resp. right) ideals of $M$ are semiprime.*

**Proof.** $\Longrightarrow$. Let $M$ be left regular, $A$ a left ideal of $M$, $x \in M$ and $\gamma \in \Gamma$ such that $x\gamma x \in A$. Then we have $x \in \Big(M\Gamma(x\gamma x)\Big] \subseteq (M\Gamma A] \subseteq (A] = A$, so $M$ is semiprime.

$\Longleftarrow$. Suppose the left ideals of $M$ are semiprime and let $x \in M$ and $\gamma \in \Gamma$. Since $(M\Gamma x\gamma x]$ is a left ideal of $M$, $x\gamma x \in M$, $\gamma \in \Gamma$ and $(x\gamma x)\gamma(x\gamma x) \in (M\Gamma x\gamma x]$, we have $x\gamma x \in (M\Gamma x\gamma x]$. Again since $(M\Gamma x\gamma x]$ is semiprime, $x \in M$, $\gamma \in \Gamma$ and $x\gamma x \in (M\Gamma x\gamma x]$, we have $x \in (M\Gamma x\gamma x]$, thus $M$ is left regular. In a similar way we prove that $M$ is right regular. □

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