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A correction to the article "On an Over-Convergence Phenomenon for Fourier Series. Basic Approach"

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In the item **4.3.3** of my work [1], the last formula of **Theorem 3** contains errors. Below is the main part of the item, including the revised proof of Theorem 1, which corresponds to Theorem 3 of [1]. The remaining numbering from [1] is preserved.

4.3.3 The quasi-polynomial representation

If the parameters $\{\lambda_k\}$ and corresponding multiplicities $\{n_k\}$ are known, then (according to (13)) we can use the representation

$$\mathfrak{t}_{r,s} = (-1)^{s-r} \Big(\prod_{\substack{p \in \mathfrak{D}_\mathfrak{n} \\ p \neq r}} \frac{s-p}{r-p} \Big) \prod_{j \in \mathfrak{D}_\mathfrak{m}} \left(\frac{r-\lambda_j}{s-\lambda_j} \right)^{n_j}, \ s = 0, \pm 1, \ldots, \tag{27}$$

where $r \in \mathfrak{D}_\mathfrak{n}$, $\mathfrak{D}_\mathfrak{m} \subset \mathfrak{D}_\mathfrak{n}$, $\{n_q\}$ are corresponding positive integers, and $\sum_{j \in \mathfrak{D}_m} n_j = n$, $\lambda_p \neq \lambda_q$ if $p \neq q$.

The following generalizes Theorem 1 for the case $\{\lambda_{r,p}\} = \lambda_p, p \in \mathfrak{D}_m$.

Theorem 1 *Suppose the sequence (27) is given. Then the corresponding functions $\{\mathfrak{T}_r\}$ are quasi-polynomials and have the following explicit form*

$$\mathfrak{T}_r(x) = \sum_{j \in \mathfrak{D}_m} \sum_{k=1}^{n_j} c_{r,j,k} \, \Lambda_{j,k}(x), \ r \in \mathfrak{D}_n, \ x \in [-1, 1], \tag{28}$$

where (see (16)) the system $\{\Lambda_{r,k}\}$ consists of the following quasi-polynomials

$$\Lambda_{j,k}(x) = \frac{-\pi}{(k-1)!} \frac{d^{k-1}}{d\,\lambda_j^{k-1}} \left(\csc(\pi \lambda_j) \exp(i \, \pi \, \lambda_j \, x) \right),$$

and

$$c_{r,j,k} = \frac{(-1)^r \prod_{p \in \mathfrak{D}_m} (r-\lambda_p)^{n_p}}{(n_j-k)! \prod_{\substack{p \in \mathfrak{D}_\mathfrak{n} \\ p \neq r}} (r-p)} \ \frac{d^{n_j-k}}{d\lambda_j^{n_j-k}} \left(\frac{\prod_{\substack{p \in \mathfrak{D}_\mathfrak{n} \\ p \neq r}} (\lambda_j - p)}{\prod_{\substack{p \in \mathfrak{D}_\mathfrak{m} \\ p \neq j}} (\lambda_j - \lambda_p)^{n_p}} \right)$$

Proof. The function $T_r(s) = \mathfrak{t}_{r,s}$, considered for $s \in \mathbb{C}$, is rational with poles of order n_j at $s = \lambda_j$, $j \in \mathfrak{D}_m$.

Let $U \subset \mathbb{C}$ be a simply connected open subset containing all points $\{\lambda_j\}$, $j \in \mathfrak{D}_m$, with the positively oriented simple boundary curve $\gamma = \partial U$. We have $T_r(s) = (1/s)$, $s \to \infty$, therefore, according to Cauchy's residue theorem

$$0 = \frac{1}{2\,\pi\, i}\int_\gamma \frac{T_r(t)}{t-s}\,dt = T_r(s) + \sum_{j\in\mathfrak{D}_m} \mathrm{Res}_{z=\lambda_j}\big(\frac{T_r(z)}{z-s}\big),\ s \in U\setminus\{\lambda_j\}.$$

Let us show how these residues can be explicitly calculated. For given $r \in \mathfrak{D}_\mathfrak{n}$ and $j \in \mathfrak{D}_m$, the problem is reduced (see (27)) to finding the residues at the point $z = \lambda_j$ for the function

$$W_1(z) = \frac{W(z)}{(z-\lambda_j)^{n_j}},\ \text{ where }\ W(z) = \frac{\prod_{\substack{p\in\mathfrak{D}_\mathfrak{n}\\ p\neq r}}(z-p)}{(s-z)\prod_{\substack{p\in\mathfrak{D}_\mathfrak{m}\\ p\neq j}}(z-\lambda_p)^{n_p}}.$$

From here

$$\mathrm{Res}_{z=\lambda_j} W_1(z) = \frac{1}{(n_j-1)!}\ \frac{d^{n_j-1}}{d\lambda_j^{n_j-1}} W(\lambda_j) =$$

$$\sum_{k=1}^{n_j} \frac{1}{(n_j-k)!\,(s-\lambda_j)^k}\ \frac{d^{n_j-k}}{d\lambda_j^{n_j-k}} \left(\frac{\prod_{\substack{p\in\mathfrak{D}_\mathfrak{n}\\ p\neq r}}(\lambda_j-p)}{\prod_{\substack{p\in\mathfrak{D}_\mathfrak{m}\\ p\neq j}}(\lambda_j-\lambda_p)^{n_p}}\right).$$

This implies (see (18) and (21)) the formula (28). □

References

[1] Anry Nersessian, On an Over-Convergence Phenomenon for Fourier Series. Basic Approach, Armen. J. Math., V. 10, N. 9 (2018), pp. 1-22.

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