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# CONFERENCE ABSTRACTS

**Normalizers of free subgroups in free Burnside groups of odd period $n \geq 1003$**

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Let $B(U, n)$ be a free periodic group of period $n$ with base $U$. In the current paper announce that for any odd $n \geq 1003$ the normalizer of any non-trivial subgroup $N$ of the group $B(U, n)$ coincides with $N$, provided that the subgroup $N$ is free in the variety of all $n$-periodical groups. From this follows a positive answer for all prime numbers $n > 997$ to the following problem set by Adjan in Kourovka Notebook: Is it true that all proper normal subgroups of the group $B(m, n)$ of prime period $n > 665$ are not free periodic groups? The current result also strengthens a similar result of Ol'shanskiy for sufficiently large odd numbers $n$ ($n > 10^{77}$).

**Topological spaces with simple clones**

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The monoid $M(X)$ of continuous maps of a topological space $X$ into itself is well known, and the relations between the algebraic structure of $M(X)$ and the topological structure of $X$ are well studied. In his book [1], W. Taylor introduced a generalization of the monoid of continuous maps — the clone $Cl(X)$ of the topological space $X$.

The clone $Cl(X)$ is a heterogeneous (multy-sorted) algebra with countably many carrier sets $C_n = \{f : X^n \to X;\ f \text{ continuous}\}$, countably many constants $\pi_i^n \in C_n$ and heterogeneous operations $S_m^n$ :

$C_n \times (C_m)^n \to C_m$ defined as follows:

$$\begin{aligned} &\pi_i^n(x_0, \ldots, x_{n-1}) = x_i, \\ &S_m^n(f; g_0, \ldots, g_{n-1})(x_0, \ldots, x_{m-1}) = \\ &\qquad\qquad f(g_0(x_0, \ldots, x_{m-1}), \ldots, g_{n-1}(x_0, \ldots, x_{m-1})) \end{aligned}$$

for any $i < n \in \omega$ and for any $m, n \in \omega$, respectively. As it could be expected, this structure carries even more information about the topological properties of $X$ than the monoid of continuous self-maps (see [2]).

In this report we will highlight some topological consequences of an algebraic property, namely simplicity, of a clone. It is known that in the class of completely regular spaces which contain a subset homeomorphic to a non-trivial interval of the real line, the clone of a topological space is simple if and only if the space is homotopically equivalent to a one-point space. The reverse is not true — the clone of any 0-dimensional space is simple (see [3]). Another example of a homotopically non-trivial non-metrizable continuum with a simple clone will be presented.

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# A New Definition of Equality of Tautologies for various Logics

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We would like to discuss a conceptual question: in what case two tautologies can be considered as equal. Let $\varphi$ and $\psi$ be propositional formulae (logical functions) and let each of them depend on the propositional variables $p_1, p_2, \ldots, p_n$. It is well-known, that $\varphi$ and $\psi$ are equal iff for every $\sigma = (\sigma_1, \ldots, \sigma_n)$ ($\sigma_i \in \{0, 1\}$, $1 \le i \le n$) $\varphi(\sigma_1, \ldots, \sigma_n) = \psi(\sigma_1, \ldots, \sigma_n)$. By this conception all classical tautologies are equal to each other. In our opinion this thesis is not entirely correct.

In fact, the tautology $\varphi_k = (p_1 \supset (p_2 \supset (p_3 \supset \ldots \supset (p_k \supset p_1) \ldots)))$ is very "simple". It is easy to notice that (i) if the value of $p_1$ is 1, then, because of its second occurrence, the value of $\varphi$ is equal to 1 without taking into consideration the values of the remaining variables, and (ii) dually if the value of $p_1$ is 0 then the value of $\varphi$ is 1 because of the first occurrence of $p_1$. So, only the variable $p_1$ is "important" in this formula, while the other variables are absolutely unimportant. In some tautologies several variables are "important", and there are also tautologies where nearly all variables are "important". It is natural, that such tautologies are "harder". In [1] the notion of $\varphi$-determinative conjunct was defined. Using this notion a new definition of equality of tautologies is suggested in [2].

Let $\varphi$ be a propositional formula and let $\{p_1, p_2, \ldots, p_n\}$ be the set of its distinct variables. For some $\sigma^m = (\sigma_1, \ldots, \sigma_m) \in E^m$ ($1 \le m \le n$) the conjunct $K = p_{i_1}^{\sigma_1} \& p_{i_2}^{\sigma_2} \& \ldots \& p_{i_m}^{\sigma_m}$ is called *$\varphi$-determinative* if the assignment of values $\sigma_j$ to each $p_{i_j}$ $(1 \leq j \leq m)$ induces the value (1 or 0) for $\varphi$, without taking into consideration the values of the remaining variables [1].

In [2] there is based one's argument on the following difinition.

**Definition.** *The classical tautologies $\varphi$ and $\psi$ are strongly equal if every $\varphi$-determinative conjunct is also $\psi$-determinative and vice versa.*

As the intuitionistic (minimal) validity is determined only by derivability in some intuitionistic (minimal) propositional calculus and $\overline{\overline{p}}$ is not equivalent to $p$ in nonclassical logic, above notion of $\varphi$-determinative conjunct is not directly applicable for these systems. The analogies of the $\varphi$-determinative conjunct for intuitionistic and minimal logics are constructed in [3].

For fuzzy logic the notion of $\varphi$-determinative conjunct is suggested by D. Alanakyan.

For some other logics the analogies of the $\varphi$-determinative conjunt can be also suggest, therefore the above definition is valid for various logics.

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## The Jordan-Postnikov Normal Form of a Real Linear Operator

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We give a direct proof of the following result: for any real linear operator there exists a basis in which it has a Jordan-Postnikov matrix. This matrix for a real linear operator is determined uniquely up to the order of direct summands on the diagonal.

## Ideals and Congruences of Cancellation Abelian Algebras

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In our talk we consider some properties of ideals and congruences of binary algebras. We fined the necessary conditions when any hyperidentity (i.e. $\forall(\forall)$-identity) is satisfed in all abelian commutative binary algebras and all abelian cancellation algebras. We proof that the lattice of ideals of finitely generated dis- tributive idempotent algebra is finite.

## Tiling the Integers with Infinite Subsets

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Let $\mathcal{N} = \{n : n = 0, 1, 2, ...\}$ and $\mathcal{Z} = \{n : n = 0, \pm 1, \pm 2, ...\}$. We only consider subsets $A \subset Z$ that are infinite and $0 \in A$. For two subsets $A, B \subset Z$ let $A \oplus B = \{a + b : a \in A, b \in B$ and if $a + b = a' + b'$ then $a - a' = b - b' = 0\}$. If $A \oplus B = \mathcal{N}(= \mathcal{Z})$, we say $B$ is a complement of $A$ in $\mathcal{N}$ (in $\mathcal{Z}$); or $A$ *tiles* $\mathcal{N}$ (tiles $\mathcal{Z}$). N.G. De Bruin in 1950 and others since, have studied the case when $A \oplus B = \mathcal{N}$. Consider the dyadic representation of $\mathcal{N}$; that is, every $n \in \mathcal{N}$ can be written as

$$n = \epsilon_0 + \epsilon_1 2 + \epsilon_2 2^2 + \cdots + \epsilon_k 2^k;$$

$\epsilon_i = 0$ or $1$ for $i = 0, 1, 2, \ldots, k$. Let $A = \{n \in \mathcal{N}$ : in the dyadic representation of $n, \epsilon_i = 0$ if $i =$ even$\}$, and $B = \{n \in \mathcal{N}$ : in the dyadic representation of $n, \epsilon_i = 0$ if $i =$ odd$\}$. Then, it is clear that $A \oplus B = \mathcal{N}$.

This can be extended to more general bases. For $i \geq 1$, let $m_i \geq 2$ be a sequence of integers; $M_0 = 1$, and $M_k = \prod_{i=1}^{k} m_i$ for $k \geq 1$. Then, in the $\{m_i\}$-adic representation, every $n \in \mathcal{N}$ can be written as

$$n = \epsilon_0 + \epsilon_1 M_1 + \epsilon_2 M_2 + \cdots + \epsilon_k M_k;$$

$0 \leq \epsilon_i < m_{i+1}$ for $0 \leq i \leq k$.

Again let $A = \{n \in \mathcal{N}$ : in the $m_i$-adic representation of $n, \epsilon_i = 0$ if $i$ = even$\}$, and $B = \{n \in \mathcal{N}$ : in the $m_i$-adic representation of $n, \epsilon_i = 0$ if $i =$ odd$\}$. Once more, $A \oplus B = \mathcal{N}$. The converse of this is also true. It can be shown that if $A \oplus B = \mathcal{N}$, then in some $m_i$-adic representation, the subsets $A$ and $B$ are obtained as described. From the above, it follows that if a subset $A$ has a complement $B$ in $\mathcal{N}$, then the subset $B$ is unique. Moreover, it should not be difficult to show that, if we remove any member of $A$, then the remaining subset will cease to tile $\mathcal{N}$. Therefore, it seems that,

- there does not exist a subset $A$ with the property that every subset $A' \subset A$ tiles $\mathcal{N}$.

In the above, replacing $\mathcal{N}$ by $\mathcal{Z}$ complicates things. It is known that if $A \oplus B = \mathcal{N}$, then the set $A$ tiles $\mathcal{Z}$, and it contains an infnite number of complements in $\mathcal{Z}$. Beyond that, I have seen little discussion on tiling $\mathcal{Z}$ with infnite subsets. I know of no algebraic arguments to clarify the situation. I study ergodic transformations on an infnite measure space. Let me describe the pertinent facts from ergodic theory that shed light on the subject.

A little of Ergodic Theory: Let $T : X \to X$ be an egodic measure preserving transformation, with $m(X) = 1$. A sequence of integers $n_i$ is an *exhausting weakly wandering sequence (eww) for* $T$ if there exists a set $W$ with $m(W) > 0$, $T^{n_i}W \cap T^{n_j}W = \emptyset$ for $i \neq j$, and $X = \cup_{i=0}^{\infty} T^{n_i}W$. The following unexpected result is true in ergodic theory.

- There exists an eww $n_i$ for $T$ such that every $n_i' \subset n_i$ is again eww for $T$.

A corollary of this result says:

- There exists a subset $A \subset \mathcal{Z}$ with the property that every subset $A' \subset A$ is a tile for $\mathcal{Z}$.

## On the Kurosh rank of the intersection of subgroups in free products and a property of groups

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The Kurosh rank of a subgroup of a free product of some groups is defined, accordingly to the classic Kurosh subgroup theorem, as the number of free factors in free decomposition of this subgroup. We prove that if two subgroups of a free product have finite Kurosh ranks, then their intersection also has finite Kurosh rank and give an upper estimate for this rank. We also introduce and study a property of groups which could be regarded as a broad generalization of the Cauchy-Davenport theorem and which turned out to be useful in the proof of the upper estimate.

## Groupification of universal algebras

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We show that many universal algebras admit group structures such that all operations can be expressed via multiplication, inverse, and constants.

## Almost path surfaces of Rieman spaces of orthogonal composition

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Consider Riemann space $V^n$, which admits a composition of two manifolds with completely orthogonal transversal positions of $V^{n_1}$ and $V^{n_2}$, $n_1 + n_2 = n$. Positions $V^{n_1}$ $(V^{n_2})$ constitute $n_1(n_2)$ parametric family of surfaces, which we call a first (second) bundle of composition.

Let $\gamma$ be almost geodesic line (see [1]) $V^n$, which do not belongs to positions $V^{n_1}$ and $V^{n_2}$. A topology product $\gamma \times V^{n_1}$ $(\gamma \times V^{n_2})$ of this line and surface $V^{n_1}$ $(V^{n_2})$ we call an outer almost path surface of a first (second) bundle. Surfaces $\gamma \times V^{n_1}$ and $\gamma \times V^{n_2}$ together we call outer almost path surfaces of composition. If $\gamma_2$ ($\gamma_1$) is an almost geodesic line $V^{n_2}$ ($V^{n_1}$), then topology product $\gamma \times V^{n_1}$ ($\gamma \times V^{n_2}$) we call an inner almost path surface of a first (second) bundle, and them together we call inner almost path surfaces of composition.

The following theorems are proved.

**Theorem 1.** *Among surfaces $X^{n_1+1} = \gamma_2 \times V^{n_1}$ only inner almost pass surfaces can be almost completely geodesic (see [2]).*

**Theorem 2.** *If the surface $X^{n_1+1} = \gamma_2 \times V^{n_1}$ is almost completely geodesic with respect to a some unit normal vector field, then at any point of the surface the vector of that field coincides with a first normal of a line $\gamma_2$ which contains this point.*

**Theorem 3.** *In Riemann space of an orthogonal composition all inner almost path surfaces of a first composition bundle are almost completely geodesic with respect to a unit normal vector field $(0, \nu_1^{\overline{a}}(u^{\overline{c}}))$ if and only if $V^n$ is semireducible space with a metrics*

$$ds^2 = e^{\rho(u^{\overline{c}})} g_{ab}(u^c) du^a du^b + g_{a\overline{b}}(u^{\overline{c}}) du^a du^{\overline{b}}$$

*Besides are connected by expression (46), where $k^\nu(u^c)$ is a some function not equal to zero at any points.*

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**Graphical script of the solutions of equations in free groups**

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In a series of G. S. Makanin papers were introduced some parameterizing functions depending on word variables, positive integer variables, and variables, whose values are finite sequences of positive-integer variables. Earlier Khmelevskij and Lindon have shown that general solution of some equations in a free monoid and in a free group did not admit so-called primitive parameterization, i.e. not every equation has general solution defined by formulae depending on variables and positive integer variables.

One can found the complete bibliography in papers of G. S. Makanin "Finite parameterization of solutions of equations in a free monoid" published in "Matematicheskij Sbornik", Volume 195, no. 2, 4, 2004.

G. S. Makanin has overcome the emerged barrier by introducing a third type variable, namely variable, whose values are finite sequences of positive-integer variables.

The present paper reports that general solution of a series of equations in a free group can be described by means of a finite number of oriented graphs, in whose vertices some invariants, prefixes of equations are marked, and on whose ribs transforms of variables are pointed. A solution set of equations will coincide with oriented graph path set.

All equations, whose general solutions can be described by means of Makanin functions $F_i$, $T_h$, $R_o$, admit also graph description using graphs of the a special type.

It also is possible to show that the solutions of the Burnside type equation $x^5y = zt^5$ in free group can be described by paths of the graph, constructed by means of subgraphs of the above mentioned graph.

## On varieties of groups generated by wreath products of abelian groups

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In this talk we outline some of our recent results about the varieties generated by wreath products $A\ Wr\ B$ of abelian groups and by wreath products $\mathcal{X}\ Wr\ \mathcal{Y}$ of arbitrary sets of abelian groups $\mathcal{X}$ and $\mathcal{Y}$.

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## Construction of infinite-dimensional relative homotopy groups in a Hilbert space

**Eduard Mirzakhanyan**
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The paper is devoted to the construction of an infinite-dimensional homotopy topology in the arbitrary real Hilbert space $H$. The paper describes constructions of infinite-dimensional analogues of finitely dimensional homotopy groups, namely, infinite-dimensional absolute and relative gomotopy groups of subsets and subset pairs from $H$. Admitted maps and homotopies are maps belonging to the special class $K_0$ and its subclass $K_0^c$ of continuous maps $f : X \to Y$ of subsets from $H$. For every of those maps abelian groups $\Pi_q(X,\, x_0)$ and $\Pi_q(X,\, A,\, x_0)$ are constructed. Imposing some restrictions such as compactness on spheroids and their homotopies we construct also groups $\Pi_q^c(X,\, x_0)$ $\Pi_q^c(X,\, A,\, x_0)$ called compact type infinite-dimensional homotopy groups. In the case of the separable Hilbert space $H$ one more equivalent approach to constructions of infinite-dimensional relative homotopy groups is found. This approach is based on the choice of the orthonormal basis in $H$.

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## Classification of compact $G$-extesions in the case of a locally compact group $G$.

**Narine Mirzakhanyan**
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The subjects of the classification of compact $G$-extensions were intensively investigated by Yu. M. Smirnov (Moscow) and J. de Vries (Amsterdam) schools. The paper is devoted to the classification of all compact $G$-extensions of the Tikhonov $G$space when acting group $G$ is locally compact. The main result is the following classification theorem.

**Theorem.** *For every invariant closed subalgebra $K \subset E^*(X)$, which separates pointes and closed sets, the pair $(\Delta_k, b_k)$, is a compact $G$-extension for the Tikhonov $G$-space $X$. Conversely, for every compact $G$-extension $(B, b)$ of the $G$-space $X$ there is an invariant closed subalgebra $K \subset E^*(X)$, which separates pointes and closed sets, and such that the pair $(B, b)$ is equivariantly equivalent to the pair $(\Delta_k, b_k)$.*

## Interlaced, modular, distributive and boolean bilattices

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In this talk we give characterizations of interlaced, modular, distributive and Boolean bilattices by hyperidentities.

**On composition of infinite-dimensional quadratic forms**

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The problem of composition of quadratic forms consists of the following: find out for which $m$ and $n$ there exist identities of type:

$$(x_0^2 + x_1^2 + \cdots + x_n^2)(y_0^2 + y_1^2 + \cdots + y_m^2) = (z_0^2 + z_1^2 + \cdots + z_n^2),$$

where $z_i$ are bilinear forms on $x_0, x_1, \cdots, x_n; y_0, y_1, \cdots, y_m$ with coefficients from a certain field $F$.

Classical Theorem of Gurwitz-Radon states that the solution of this problem consists of all pairs $(m, n)$, where $n$ is an odd number and $\rho(n) \geq m \geq 0$ (here $\rho(n)$ is the is the number of Gurwitz-Radon).

Existence of compositions of quadratic (not necessarily finite-dimensional) forms is closely related to the problem of existence of compositions, and in particular, normed algebras.

Kaplanski showed in [1] that there does not exist infinite-dimensional normed algebras $A$ with a unite element on a field $F$, except the case when $A$ is the purely non-separable field over $F$, and is of characteristic 2. Based on this he posed a hypothesis, that irreducible infinite-dimensional quadratic forms do not possess composition, except the trivial case of the purely non-separable fields of characteristic 2.

This hypothesis is not true: based on the work [2] the author builds compositions of an infinite-dimensional positively defined quadratic form (work in print).

Let us notice for a sake of comparison that the infinite-dimensional real normed algebras appearing in our construction have a left unity only. Also, in these algebras left shifts always are bijective, and right shifts never are surjective.

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## On the products of $T$-ideals in free associative algebras

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Let $F$ be a field and $A$ a free associative $F$-algebra or a group algebra of a free group with an infinite set $X$ of generators. We find a necessary and sufficient condition for the inclusion $I' \subset I$, where $I = I_1 \cdots I_k$ and $I' = I'_1 \cdots I'_l$ are any products of $T$-ideals in $A$. A canonical reformulation in terms of products of group representation varieties answers a question posed in 1986.

## Axiomatization of the group generated by point-reflections of metric planes with non-Euclidean metric

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We provide an axiom system for the group of motions generated by the point-reflections of non-elliptic metric planes with non-Euclidean metric. The axiom system is expressed in terms of motions and the operation of composition of motions. Given that it is not known whether there exists an integer $k$ such that each product of point-reflections can be written as a product of at most $k$ reflections, the first-order theory of this group is not known. The only possible axiomatization with our present knowledge is an infinitary one.

## Connection on the Space of Linear Elements

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Let $G$ is a homothetic group of strips and $\nabla$ is a linear connection on tangent bundle $TM$. Then the factor $TM = LM/G$ is the space of linear elements.

If

a)affin or of tangential structure is co variable constant,

b)affin or of almost product is also co variable constant,

c) $\nabla$ is invariant with respect to group $G$,

d)infinitesimal connection is defined by the given connection $\nabla$,

then $\nabla$ is $G$-projectile and a projection of this connection is the connection on the space of linear elements.

## Logically geometrical equivalence of two algebras

**Boris Plotkin**
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We consider algebras in a variety of algebras $\Theta$. A well known invariant of every algebra $H \in \Theta$ is its elementary theory $Th(H)$. Two algebras $H_1$ and $H_2$ are elementary equivalent if $Th(H_1) = Th(H_2)$. We introduce a more strong notion of logically geometrical equivalence of two algebras ($LG$-equivalence). This $LG$-equivalence implies elementary equivalence, but not vice versa.

In the talk we consider problems related to the notion of $LG$-equivalence of algebras. In particular, let us mention the following one:

Let $\Theta$ be an arbitrary variety of algebras, $W = W(X)$ a free algebra in this variety with the finite set $X$. We say that this algebra $W$ is $LG$-separable in $\Theta$, if any other algebra $H$, $LG$-equivalent to $W$, is isomorphic to $W$. It is proved that this property holds for free semigroups and free inverse semigroups. A study of other interesting cases is in progress.

## Baer-Suzuki-type theorems in solvable case and around

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The Baer-Suzuki theorem characterizes the nilpotent radical of a finite (or linear) group by the property: an element $g$ is in the nilpotent radical of $G$ if and only if any two conjugates of $g$ generate a nilpotent group.

We discuss a general setting whose part is the following sharp analog of the Baer-Suzuki theorem for the solvable radical of a finite (linear) group: an element $g$ is in the solvable radical of $G$ if and only if any four conjugates of $g$ generate a solvable group.

This result is independently announced by Flavell, Guest, Guralnick.

In the talk we outline the proof of this theorem. We also give a sketch of the proof of the weaker result: an element $g$ is in the solvable radical of $G$ if and only if any seven conjugates of $g$ generate a solvable group.

Joint work with N.Gordeev, F.Grunewald, B.Kunyavskii

## Automorphic equivalence of multimodels recognition

**Marina Knyazhansky**
**Tatyana Plotkin**
Bar Ilan University, Tel Aviv, Israel

The talk is the continuation of the talk "Automorphic equivalence of algebras and bases of algebraic knowledge" which reduces the problem of informational equivalence of knowledge bases to the one of recognition of automorphic equivalence of multimodels. The notion of informational equivalence of knowledge bases is defined algebraically. However this formal definition is not constructive. Hence, for a practical verification of informational equivalence of knowledge bases one has to look for a constructive algorithm. In this talk we build such an algorithm for two cases: the sets $A$ and $B$ are finite (one-sorted for simplicity) sets of $n$ elements, and $A$ and $B$ are vector (linear) spaces over a finite field of $p$ elements.

Even in the general case we can show that this problem can be reduced to the situation when the set of relations $\Phi$ consists of one universal relation. The arity of the relation $\Phi$ depends on $n$ in the first case and on the dimension $p$ of the space in the second one.

We show that in the first case the problem is reduced to the problem of verification if two subgroups in a symmetric group $S_n$ are conjugated. In the second case it is reduced to the problem of conjugacy of two subgroups in the general linear group $GL_n(p)$.

## Automorphic equivalence of algebras and bases of algebraic knowledge

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In the talk we define the notion of automorphic equivalence of two models. Knowledge bases are built over these models. Our main destination is to figure out what are the relations between the models that provide the informational equivalence of the corresponding knowledge bases.

The main result is as follows: *knowledge bases are informationally equivalent if and only if the corresponding models are automorphically* *equivalent*. This result motivates the importance of the notion of automorphic equivalence of the models. We assume that all multimodels are finite.

A model is considered as a triple $(A, \Phi, f)$, where $A$ is an algebra in a variety $\Theta$, $\Phi$ is a set of symbols of relations, and $f$ is an interpretation of all these symbols in the algebra $A$. In fact, we will consider a multimodel $(A, \Phi, F)$, where $F$ is a set of interpretations. This multimodel induces a knowledge base $KB = KB(A, \Phi, F)$, which is a base of our knowledge about the multimodel $(A, \Phi, F)$.

Given two models $(A, \Phi_1, f_1)$ and $(B, \Phi_2, f_2)$, denote their groups of automorphisms by $Aut(f_1)$ and $Aut(f_2)$, respectively. Each of them consists of automorphisms of the algebra, correlated with the given relations. These two models are called *automorphically equivalent* if there exists an isomorphism $\delta : A \to B$ such that $Aut(f_2) = \delta Aut(f_1)\delta^{-1}$. Multimodels $(A, \Phi_1, F_1)$ and $(B, \Phi_2, F_2)$ are automorphically equivalent if there exists a bijection $\alpha : F_1 \to F_2$, such that for every $f \in F_1$ the models $(A, \Phi_1, f)$ and $(B, \Phi_2, f^\alpha)$ are automorphically equivalent. Automorphic equivalence of algebras is a particular case of this general definition. Besides, algebras $A$ and $B$ may be multisorted in view of applications.

Given a multimodel $(A, \Phi, F)$, we build an algebraic model of the corresponding knowledge base. Such a model allows us to give a precise definition of informational equivalence of two knowledge bases. We distinguish three components of knowledge: the description of knowledge (syntax), the subject of knowledge (applied field) and the content of knowledge (semantics). Accordingly we consider the categories. The first one is the category of logical description of knowledge, denoted by $LD_{\Phi\Theta}$. Its objects have the form $(X, T)$, where $T$ is a set of logical formulas, written using variables from the set $X$. The morphisms are defined as well.

The categories presenting the content of knowledge are denoted by $CK_{\Phi\Theta}(f)$, for all $f \in F$. Their objects have the form $(X, A)$, where $A$ is a set in an Affine space, determined by a set of formulas $T$. There is a functor between these two categories: $Ct : LD_{\Phi\Theta} \to CK_{\Phi\Theta}(f)$. We treat such triples (functor and two categories) running all $f \in F$ as a *knowledge base* $KB(A, \Phi, F)$.

Consider two multimodels $(A, \Phi_1, F_1)$ and $(B, \Phi_2, F_2)$, and take the corresponding knowledge bases $KB_1$ and $KB_2$. Let a bijection $\alpha : F_1 \to F_2$ be given. Suppose there exist the homomorphisms of categories $\beta$ from $LD_{\Phi_1\Theta}$ to $LD_{\Phi_2\Theta}$, $\beta'$ in the opposite direction, and an isomorphism of categories $\gamma : CK_{\Phi_1\Theta}(f) \to CK_{\Phi_2\Theta}(f)$ subject to natural correlation conditions. Then the knowledge bases $KB_1$ and $KB_2$ are called *informationally equivalent*.

This definition means that everything which can be obtained from $KB_1$ can be also obtained from $KB_2$ and vice versa. The formulated theorem claims that knowledge bases are informationally equivalent if and only if the corresponding multimodels are automorphically equivalent. This means that a verification of an informational equivalence of knowledge bases can be reduced to checking whether the corresponding multimodels are automorphically equivalent.

## Grobner Bases and Local Classification Problems in Analysis According to Arnold

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V. I. Arnold in his papers ([1], problem 1994-24 and [2]) proposed to study rationality of Poincare series for so called local classification problems of analysis. It turns out that for arbitrairy pseudogroup action always exist a strata consisting of finite number of open sets (atoms) with following property: any two points belonging to the same atom have the same Poincare series and Tresse theorem is valid for each point of this strata. We call this strata the maximal strata; this strata is an open dence set in the corresponding infinite jet space. If the defining relations of pseudogroup are analytic functions, the maximal strata always consists of a single atom and is a complement to intersection of suitable hypersurfaces. The situation is more complicated outside of maximal strata. As an example of local classification problem let us consider the problem of classification of vector fields at a neighborhood of a fixed point of the manifold (example 2 of [1]). In this local classification problem the maximal strata consists of vector fields, which do not vanish at a fixed point. There exists a set, which is open in the complement to the maximal strata and in this set the subset of points with non-rational Poincare series is dense. Every local classification problem of analysis corresponds to the action of its own diffeomorphism group, i.e. Lie pseudogroup. This allows one to derive the existence of maximal strata and Tresse theorem from the result mentioned above. Among other results we emphasize the following ones:

- In principle one can explicitely derive conditions defining each atom from Lie pseudogroup equations (using finite number of differentiations and algebraic operations).
- In principle for each atom the corresponding Poincare series can be computed explicitely.
- In Tresse theorem basic invariants and invariant differentiations for each point of the maximal strata in principle can be computed explicitely (using differentiations, algebraic operations and solution of systems of algebraic equations via implicit function theorem).

References

[1] Arnold V.I., Mathematical problems in classical physics, Trends and Perspectives in Applied Mathematics (Appl.Math.Series,V.100), Springer, 1994, 1-20.

[2] Arnold problems (in Russian), M., Fasis, 2000.

## On Magnus Groups

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Consider the variety of all groups which are extensions of abelian groups by nilpotent groups of class les than $C$. The main result belonging to Sirtsov states that this variety is strongly Magnus, that is, free products (inside this variety) of Magnus groups are Magnus groups.

The result was obtained by D. I. Eidelkind for $c = 2$. He also proved that this is no longer true for other polynilpotent varieties.

## Algebraic groups and Lie groups with few factors

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Algebraic groups are treated here from a group theoretical point of view and the obtained results are compared with the analogous issues in the theory of Lie groups. The main body of the text is devoted to a classification of algebraic groups and Lie groups having only few subgroups or few factor groups of different type. In particular, the diversity of the nature of algebraic groups over fields of positive characteristic and over fields of characteristic zero is emphasized. This is revealed by the plethora of three-dimensional unipotent algebraic groups over a perfect field of positive characteristic, as well as, by many concrete examples which cover an area systematically. In the final section, algebraic groups and Lie groups having many closed normal subgroups are determined.

## About most Convex Bodies and Continua

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I will present generic properties of convex bodies and continua. These will include boundary properties (curvature, geodesics), global properties, and results about the efforts to pass through small holes.